

# Linear Systems and $k$ -Dimensional Weisfeiler-Lehman Stabilization

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## *ABSTRACT*

In our application, we desire to canonically order nodes of an unmanned vehicle swarm. If nodes are randomly distributed, the PageRank algorithm used in some search engines to order query responses often yields a canonical ordering. Such a canonical ordering uniquely identifies a graph and can be used to decide graph isomorphism. If the PageRank algorithm fails to yield a canonical ordering, algorithms that decide graph isomorphism by finding a canonical ordering may be applied instead. For instance, *nauty* is the traditional discrete algorithm for computing a canonical vertex ordering. A key step in that algorithm is to find an equitable partition, which is equivalent to the more general concept of finding a 1-D Weisfeiler-Lehman stabilization.

We first review finding a 1-D and 2-D Weisfeiler-Lehman stabilization using discrete and linear algebraic methods, along with the Perron-Frobenius and Gershgorin Circle Theorems. We recall our construction for ensuring the matrix inverse exists and demonstrate it can also be used to find 1-D and 2-D Weisfeiler-Lehman stabilizations. This construction yields the main result, an algorithm that uses certain linear systems to compute a  $k$ -D Weisfeiler-Lehman stabilization. To our knowledge, this simple algorithm is the first linear algebraic method for computing a  $k$ -D Weisfeiler-Lehman stabilization. We conclude with techniques for reducing the impact of using finite floating-point precision and increasing the algorithm's efficiency.