Determining a Canonical Graph Isomorph Using Lexicographic Sorting

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We are interested in canonically ordering multi-dimensional data such as node positions of an unmanned aerial vehicle swarm. Assuming the data is represented by a pair-wise distance matrix, this problem is also equivalent to determining graph isomorphism. Our approach to finding a canonical ordering is based on the PageRank algorithm, which sorts vertices, such as web pages, on the leading eigenvector of a perturbed adjacency matrix. This eigenvector exists and is unique by the Perron-Frobenius theorem, but the ordering it yields is typically not canonical.

The algorithm we present, IsoCanon, orders vertices by lexicographically sorting on the inverse of the graph's adjacency matrix. Since the inverse of an adjacency matrix may not exist, we use two isomorphism-preserving perturbations that apply the Gershgorin Circle theorem to guarantee the inverse exists and that it can be efficiently obtained. This simple algorithm terminates within a logarithmic set of iterations relative to the vertex set magnitude and yields canonical orderings of many tested random and regular graphs.

To increase IsoCanon's ability to find a canonical ordering, we leverage the necessary condition that vertices in the same orbit share identical inverse entries. Thus, the lexicographic sorting step is modified to sort first on individually sorted rows, and then the native rows of the inverse. A more advanced use of this condition enables IsoCanon to identify more difficult graphs, such as Paley graphs, many posed by Mathon, ladder graphs, and some Hadamard-based graphs.

IsoCanon has three key features that are useful for sorting multi-dimensional data: it terminates in polynomial time, canonically orders many graphs, and this ordering is akin to PageRank with respect to connectivity and vertex significance. IsoCanon uses parallel numerical libraries to obtain the inverse and has been extensively tested on dense graphs having up to 4,000 vertices.